A. Makhlin

Department of Physics and Astronomy, Wayne State University, Detroit, MI 48202

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We consider a Euclidean extension of the wedge form of Hamiltonian dynamics, which explicitly accounts for the strong localization of the first interaction in nuclear collisions. A new principle of the analytic continuation via the tetrad vector is introduced. We discover the existence of self-dual solutions with short life-times (ephemerons) and conjecture that these vacuum fluctuations can lower the Euclidean action of the system of the colliding nuclei, thus enforcing a breakdown of the nuclei coherence. We suggest that the ephemerons can be identified with the gluons-partons, which are resolved in high-energy nuclear collisions.

I. INTRODUCTION

In this paper, we continue our study of the scenario of ultrarelativistic heavy ion collisions. By scenario we mean the continuous temporal sequence of stages, smoothly developing one into another. These stages are different only in the respect, that each of them is characterized by its individual *optimal set* of normal modes. Posed in this way, the problem of the scenario falls out of the jurisdiction of scattering theory, and makes a powerful formalism of the S-matrix inapplicable. Instead, we suggested to use the formalism of quantum field kinetics (QFK) [1] and to treat the scenario as a problem of an inclusive measurement. In Ref. [2] and the previous papers of this cycle [3–6] (further quoted as papers [I - IV]), we focused on different aspects of the scenario. We began, in Ref. [2], with the issue of the temporal evolution of quantum fluctuations consistent with a given inclusive probe and revealed its identity with the well-known QCD evolution. This perspective has been broadened in paper [I] and brought us to the conclusion that the dense system of quarks and gluons, which is commonly associated with the quark-gluon plasma (QGP), can be formed only *in a single quantum transition*. This conclusion is based on the fact that the scale of the entire process is associated with the properties of the final state. In papers [II] and [III], we studied the geometrical properties of the collision process, concentrating on the localization of the initial interaction, which is a consequence of the *finite size* of the colliding nuclei. The Lorentz contraction confines the whole process within the past and the future domains of their intersection. The existence of the *macroscopic* light-cone limits was accepted as the classical boundary condition for all quantum fields that emerge in the collision process, leading to a new *wedge form* of Hamiltonian dynamics, which employs the proper time τ as a Hamiltonian time of the evolution. Finally, in paper [IV], using the framework of the wedge dynamics, we computed the effective mass of a soft quark propagating through the expanding background of hard partons. The leading interaction, which is responsible for the effective mass, appeared to be the chromo-magneto-static interaction of the color currents flowing in the rapidity direction. In paper [II], such currents were shown to be an intrinsic property of the states of wedge dynamics.

That part of the study was motivated by the problem of entropy production in ultrarelativistic nuclear collisions. Namely, we had to find a basis of states which is suitable for the explicit computation of the entropy. It turned out, that the states of this basis are formed gradually. The method of QFK which, in its essence, is the calculus of time-dependent Heisenberg observables, has been designed as an *ad hoc* tool adequate to that part of the problem. The

QFK reduces the problem of an inclusive measurement[1] to the study of the possible field configurations (fluctuations) that could develop before the measurement and are compatible with a given probe. Thus, this is a problem of real-time fluctuations limited by some observable at the end of the evolution, and driven by pQCD. In order to comply with the causality principle, these unobserved fluctuations must be renormalized in such a way, that *before* the localized interaction of the inclusive measurement indeed happens, the decomposition of the nuclei in terms of the final-state modes can be only virtual. The main object, that QFK is dealing with, is the inclusive cross section (as compared to the exclusive amplitude of the S-matrix formalism). The integral equations of the QCD evolution sum all the *exclusive probabilities* over a complete set of the unobserved *states*. Therefore, the perturbative expansion of the inclusive cross section includes the on-mass-shell correlators corresponding to these *states*. As it was shown in papers [2,3], their presence allows one to establish the temporal order of the accomplished subprocesses.

The nature of the mechanism that is responsible for the initial breakdown of the nuclei coherence is not yet clarified. We do not know: how the QCD interactions that make color charges visible are suddenly switched on, if there were any time dependent fluctuations of color charges before the nuclei has physically intersected,[2] and what is the geometry of the primordial colored objects. The most popular parton model (in all its modifications) implicitly relies on the impulse approximation and a time delay in the infinite momentum frame, which does not seem to be fully consistent with the picture of the gradually formed final states. In this paper, we put forward a hypothesis that can lay the footing for a more detailed study of this issue.

Our key observation is that before the moment in time when the nuclei overlapped geometrically, nothing except the nuclei themselves can be qualified as quantum states, and all conceivable fluctuations should be treated as the classical fields that coherently add up and form the two nuclei. Hence, addressing the QCD evolution before this moment, we must study not the inclusive probability,

$$\mathcal{P}(A_i + B_i \to a_f + X) = \sum_X |\langle X|a_f S A_i^\dagger B_i^\dagger|0\rangle|^2,$$

which sums up various contributions from the physical final states, but the inclusive amplitude,

$$\mathcal{S}(A_i + B_i \to a_f) = \int dxdydz \varphi_f(x)\Delta^{-1}(x)\Phi_A(y)\Phi_B(z) \ll \int e^{iS}\varphi(x)J_A(y)J_B(z)\mathcal{D}A\mathcal{D}\psi\mathcal{D}\bar{\psi} \gg,$$

where $\varphi_f(x)$ is the wave function of the inclusive probe and the operator $\Delta^{-1}(x)$ has appeared as the result of the LSZ reduction. The wave functions $\Phi_{A,B}$ correspond to the initial hadronic states and the phenomenological vertices $J_{A,B}$ connect them with the quark and gluon fields. The functional integration can be effective only if the newly emerging fields have a finite action and can be treated semi-classically in Euclidean space. Then, the symbol $\ll \cdots \gg$ stands for the average over an ensemble of these classical fields. As a matter of fact, we are looking for a special kind of Euclidean vacuum fluctuations which could be active in the closest proximity of the collision point and become pure gauge fields outside an actual "reaction zone". We expect that these fluctuations, being added to the already known ones, will lower the Euclidean action and force the conversion of the nuclei into a collective quark-gluon system. In order to pose this problem in a consistent way, we must create a framework, where the final states of the QCD evolution (previously treated in the scope of QFK) will have a recognizable image among the other Euclidean fluctuations and could be

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[1] As we emphasized in paper [II], the collective interactions in the final state can be viewed as an inclusive measurement. QGP is a collective detector for the quark and gluon modes after the first interaction in nuclear collision.

[2] In the case of two neutral atoms bound by electromagnetic forces, the time-dependent fluctuations of the charge density do exist and lead to the van-der-Vaals interaction between the atoms. This kind of interaction between the hadrons is not known.

compared with instantons [3]. An example of such a *transient* fluctuation is described in Sec. III. We indeed find the Euclidean fluctuations, which have the following properties:

(A) They can “compete” with the well known fluctuations (instantons) in the Euclidean vacuum which provide the integrity of the isolated hadrons and nuclei;

(B) They can be resolved only due to the specific geometry of the collision.

Acting in this way, we may hope to bridge the gap between the QFK and the S-matrix scattering theory, and to approach the problem of the transient process, as it was posed in the papers [I] and [II], from the Euclidean side. These two approaches are not mutually exclusive, they just answer different questions. Indeed, the QFK allows one to study how the final states (which can be *defined* only in Minkowski space) are formed. Then, these states can be continued to the Euclidean space as fluctuations, and one can ask if the presence of these fluctuation can lower the action of the whole system. To make the discussion more consistent, we must specify the environment where one can discuss the conditions (A) and (B).

A. The Euclidean field theory.

The most important thing one should remember when addressing Euclidean field theory (EFT) is that the EFT is just the S-matrix scattering theory, rewritten in a specific way. We have to explain this point in some detail, since in most of the modern applications of EFT, it is shaded. The design of EFT includes several steps:

1. The S-matrix amplitude is written down in Minkowski space. This is unavoidable, since only in this way can we define the initial and final *states*. (The states must be normalized on some space-like hyper-surfaces which exist only in the Minkowski world. Neither S-matrix, nor the LSZ reduction formulae can be derived in Euclidean space.)

2. After an S-matrix element is transformed into the momentum representation, the integrals over the real energies corresponding to the internal momenta k^μ are identically transformed to the integrals over the imaginary axis, $k_M^0 \to ik_E^0$. This procedure is known as Wick rotation. It moves the integration path away from the poles located near the real axis. Practical convenience of this strategy of computing the scattering amplitudes is the improved convergence of the integrals. The Wick rotation is possible solely because the poles of the T-ordered Feynman propagators are located in the second and fourth quadrants of the plane of complex energy. In the course of Wick rotation, the integration path never crosses the poles. Clearly, this step is not an analytic continuation. If the external momenta p^μ are considered off-mass-shell, then the external energies become the independent arguments. The Feynman amplitude is computed on the imaginary axis of energy p^0, being analytically continued to real axis at the end of calculations.

3. The next step is to introduce an auxiliary “coordinate representation” by means of the formal Fourier transform from the Euclidean momenta to Euclidean coordinates. Acting in this way, we translate the oscillating Minkowski propagators into their exponential Euclidean counterparts (thus preserving the previously gained advantage of rapid convergence). The new object still belongs to the S-matrix theory in Minkowski space. This is the same matrix element labeled by the quantum numbers of in- and out-particles. Finally, we can formulate a set of mnemonic rules (resembling the rules of a real field theory) that allow one to directly generate the results of the previous mathematical

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[3] In this paper, we accept the point of view that the properties of hadrons can be described by the “propagation” of quark correlators through the instanton liquid [7], since this approach relies on the most direct implementation of the least action principle. Perhaps, not every reader will take this for granted. A conceivable alternative (which is closer to the standard OPE-based approach [8]) would be to study the competition between the transient field configurations and the stationary QCD condensates.

manipulations and avoiding explicit reference to the Minkowski space. This is what we eventually call Euclidean field theory. At the classical level, it includes the new metric, new definitions of vectors and spinors, new action, new equations of motion that minimize the new action, etc. In any of these representations, we can pass over to the path-integral representation of the truncated amplitudes, and integrate over the field configurations that satisfy the artificial Euclidean equations of motion. An important new element that is found in the context of the Euclidean QCD is that the auxiliary Euclidean equations of motion have topological solutions, instantons, which have no real analog in the Minkowski world. However, these objects make a quantitative description of the properties of stable hadrons possible [9].

A few remarks are in order:

1. Quantum mechanics strictly prohibits any observation of the intermediate dynamics of an exclusive scattering process. None of the intermediate Minkowski momenta or Minkowski coordinates, (as well as Euclidean momenta or Euclidean coordinates) can be measured. There is no "earlier" and "later" in the Euclidean coordinate picture in exactly the same way as there are no earlier and later in Minkowski description of the interior of the S-matrix amplitude.

2. Up until now, the instanton-based calculations proved to be successful in a relatively narrow class of problems, which can be reduced to the forward scattering (i.e., propagation) of a single hadron. In this case, we can connect the physical hadronic observables and the QCD degrees of freedom using the phenomenological quark-hadron vertices (Ioffe currents). Though it may be very difficult to do it practically, there is no doubt, that the method must also describe the forward scattering of a single nucleus.

3. The Ioffe currents are color neutral and there is no colored observables in the entire problem. This fact is remarkably reflected in the unique polarization properties of instanton fluctuations; Nature "hides" the color in a very elegant way, by identifying directions in the color space with the directions in the non-observable Euclidean geometric background. Once again, this is a consequence of the quantum mechanical veto on any intrusion into the intermediate stage of an exclusive scattering process.

4. None of the problems of real-time evolution, where causality prescribes a certain order of interactions, e.g. the temporal dynamics of inclusive processes, can be reduced to the EFT. Formally, the retarded propagators just do not allow for the Wick rotation. Physically, the establishing of a certain temporal order requires some measurement.[4] As a consequence, the inclusive measurements (including the evolution equations that describe these measurements) cannot be related to the instanton physics.

B. Geometry of the collision and the field theory.

The last conclusion of the previous section may sound too pessimistic, since it seems to leave no hope for the establishing any connection between the high-energy processes and the non-perturbative physics of the hadronic world. We argue that this connection can be re-established due to the finite size of the colliding nuclei, which allows one to impose classical boundary conditions on the dynamics of *all* quantum fields. These new boundary conditions create a framework of the wedge dynamics (proposed in papers [I] and [II]). This approach was designed in order to solve the problem of the evolution of observables (like the inclusive distributions of partons-plasmons) in ultrarelativistic

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[4] A detail discussion of the connection between an observable and the temporal order in the inclusive process is given in paper [I] of this cycle.

nuclear collisions and to give a physically motivated definition of the final states in this process. Now we argue that the previously found final states (corresponding to a strongly localized interaction of the two nuclei) can be used to pose an S-matrix scattering problem. The latter can be translated into the path-integral language, and one can look for the field configurations that minimize the action of the Euclidean wedge dynamics. To ensure the applicability of the wedge dynamics in the theoretical analysis of the scattering problem, one has to select an ensemble of events with the widest rapidity plateau. The properties of the physical states in this dynamics were studied in papers [II] and [III].

In order access the Euclidean domain of nuclear collisions, we shall keep in mind an hypothetical S-matrix amplitude in which the connection between the initial states and the fundamental fields of QCD is provided by something like giant Ioffe currents of the multi-nucleon systems. Since we aim at the scenario of a nuclear collision, we must view the intermediate collective modes of the expanding quark-gluon matter as the final states. Using these states, we can pose a scattering problem, which can be then analytically continued to Euclidean variables and given a path-integral representation. It is plausible that, in the first approximation, the problem still can be reduced to the "propagation" of the quark currents through an ensemble of the Euclidean configurations of the gluon field. In wedge dynamics, the external variables of this problem are the rapidities of the incoming nuclei, and the rapidities and the transverse momenta of the final state modes. (As we shall see later, the rule of correspondence between the Minkowski and Euclidean rapidities reads as $\theta_M \to i\theta_E$.) In this geometrical context, we will be able to describe propagation of *two nuclei* using *the same Euclidean variables.* Furthermore, we indeed will find a new class of self-dual solutions that minimize the Euclidean action and are active only in the closest proximity of the collision center. This result opens an opportunity to approach the binding interactions in the nuclei and the high-energy process of the breakdown of their coherence from the same point of view.

The rest of the paper is organized as follows. In Sec. II, we review the geometric background of the wedge dynamics in Minkowski space and pass over to its Euclidean counterpart by means of the analytic continuation of the time-like tetrad vector. In Sec. III, we explicitly find the self-dual solutions of the Euclidean wedge dynamics that appear to be the short-lived eigenstates of the $SU(2)$ color matrix σ^3 (ephemerons). In Sec. IV, we return to the previously found Minkowski solutions and establish a full set of rules of the analytic continuation. Finally, in Sec. V, we compute the Euclidean action of the ephemeron and show that its topological charge is connected with the Thomas precession of its spin. We conclude in Sec. VI.

II. THE INTERNAL GEOMETRY OF THE ULTRARELATIVISTIC NUCLEAR COLLISION

The only tool which is capable of coping with the colored gauge fields in the curved geometry is the so-called tetrad formalism (see, e.g., [10,11]). Indeed, the vector and spinor fields are essentially defined in the tangent space. In a tetrad basis, components of any tensor (e.g $A^\alpha(x)$, γ^α) become scalars with respect to a general coordinate transformations and behave like Lorentz tensors under the local Lorentz group transformations. The usual tensors are then given by the tetrad decomposition, $A^\mu(x) = e^\mu_{\ \alpha}(x)A^\alpha(x)$, $\gamma^\mu(x) = e^\mu_{\ \alpha}(x)\gamma^\alpha$, etc. The covariant derivative of the tetrad vector includes two connections (gauge fields). One of them, the Levi-Civita connection

$$\Gamma^\lambda_{\ \mu\nu} = \frac{1}{2}\mathrm{g}^{\lambda\rho}\left[\frac{\partial \mathrm{g}_{\rho\mu}}{\partial x^\nu} + \frac{\partial \mathrm{g}_{\rho\nu}}{\partial x^\mu} - \frac{\partial \mathrm{g}_{\mu\nu}}{\partial x^\rho}\right] ,$$

is the gauge field which provides covariance with respect to the general transformation of coordinates. The second gauge field, the spin connection $\omega_\mu^{\ \alpha\beta}(x)$, provides covariance with respect to the local Lorentz rotation.

The curvilinear metric of wedge dynamics in Minkowski space,

$$ds^2_{_M} = -d\tau^2_{_M} + \tau^2_{_M} d\eta^2_{_M} + dr^2 + r^2 d\phi^2 \ , \tag{2.1}$$

corresponds to the parameterizations of the flat Minkowski space according to

$$\begin{aligned} x^0 &= \pm\tau \cosh\eta \ , \quad x^3 = \pm\tau \sinh\eta \ , \\ x^1 &= r\cos\phi \ , \quad x^2 = r\sin\phi \ , \end{aligned} \tag{2.2}$$

where $x^\mu = (\tau, r, \phi, \eta)$ are the contravariant components of the curvilinear coordinates that cover the future (plus sign in Eq. (2.2)) and the past (minus sign) of the hyperplane $t = 0, \ z = 0$ of the interaction, and $x^\alpha = (t, x, y, z) \equiv (x^0, x^1, x^2, x^3)$ are the Cartesian coordinates of the Minkowski space.[5] For the coordinates (2.2), the four tetrad vectors $e^\alpha_{\ \mu}$ form the matrices

$$e^\alpha_{\ \mu} = \mathrm{diag}(1, 1, r, \tau) \ , \quad e_\alpha^{\ \mu} = \mathrm{diag}(1, 1, r^{-1}, \tau^{-1}) \ . \tag{2.3}$$

These vectors correctly reproduce the curvilinear metric $\mathrm{g}_{\mu\nu}$ and the flat Minkowski metric $g_{\alpha\beta}$, *i.e.*,

$$\begin{aligned} \mathrm{g}_{\mu\nu} &= g_{\alpha\beta} e^\alpha_{\ \mu} e^\beta_{\ \nu} = \mathrm{diag}[-1, 1, r^2, \tau^2] \ , \\ g^{\alpha\beta} &= \mathrm{g}^{\mu\nu} e^\alpha_{\ \mu} e^\beta_{\ \nu} = \mathrm{diag}[-1, 1, 1, 1] \ . \end{aligned} \tag{2.4}$$

The spin connection can be found from the condition that the covariant derivative of the tetrad vectors is equal to zero [11],

$$\nabla_\mu e^a_{\ \nu} = \partial_\mu e^a_{\ \nu} + \omega_\mu{}^a{}_b e^b_{\ \nu} - \Gamma^\lambda_{\ \mu\nu} e^a_{\ \lambda} = 0 \ , \tag{2.5}$$

which can be solved for the components of ω,

$$\omega_\mu^{\ \alpha\beta} = [\Gamma^\lambda_{\ \mu\nu} e^\alpha_{\ \lambda} - \partial_\mu e^\alpha_{\ \nu}] e^{\beta \ \nu} \ . \tag{2.6}$$

Indeed, the tetrad vector $e^\alpha_{\ \mu}$ is the coordinate vector and the Lorentz vector at the same time. (The Lorentz index α and the coordinate index μ are moved up and down by the local Minkowski metric tensor $g_{\alpha\beta}$ and the global metric tensor $\mathrm{g}_{\mu\nu}$, respectively.) The only non-vanishing components of the connections are

$$\begin{aligned} \Gamma^{\cdot}_{\eta\eta\tau} &= -\Gamma^{\cdot}_{\tau\eta\eta} = -\tau, \ \Gamma^{\cdot}_{\phi\phi r} = -\Gamma^{\cdot}_{r\phi\phi} = -r \ , \\ \omega_\eta^{\ 30} &= -\omega_\eta^{\ 03} = 1, \ \omega_\phi^{\ 12} = -\omega_\phi^{\ 21} = -1 \ . \end{aligned} \tag{2.7}$$

The metric of the "Euclidean" counterpart of the wedge dynamics is,

$$ds^2_{_E} = +d\tau^2_{_E} + \tau^2_{_E} d\eta^2_{_E} + dr^2 + r^2 d\phi^2 \ . \tag{2.8}$$

In the tetrad formalism, the transition to the Euclidean space is easily done by making the time-like tetrad vector $e^0_{\ \mu}$ imaginary, $e^0_{\ \mu} \to (e^0_{\ \mu})_{_E} = (i, 0, 0, 0)$, $\ (e_0^{\ \mu})_{_E} = (-i, 0, 0, 0)$. Then Eqs. (2.4) take the form

[5] We choose the polar coordinates in the xy-plane, since we are interested in highly localized objects capable of initiating a large p_t-transfer.

$$
\begin{aligned}
\mathrm{g}_{\mu\nu} &= g_{\alpha\beta}({e^{\alpha}}_{\mu})_E({e^{\beta}}_{\nu})_E = \mathrm{diag}[1,1,r^2,\tau^2]\ , \\
g^{\alpha\beta} &= \mathrm{g}^{\mu\nu}({e^{\alpha}}_{\mu})_E({e^{\beta}}_{\nu})_E = \mathrm{diag}[-1,1,1,1]\ .
\end{aligned} \tag{2.9}
$$

This formal step also leads to a set of standard prescriptions for the transition to the Euclidean version of the field theory, like $A_E^\tau = ({e_0}^\tau)_E A^0 = -iA^0$ and $\gamma_E^\tau = ({e_0}^\tau)_E\gamma^0 = -i\gamma^0$. The same rule holds for the spin connection,

$$
({\omega_\mu}^{03})_M \ \to\ ({\omega_\mu}^{03})_E = -i({\omega_\mu}^{03})_M. \tag{2.10}
$$

These formulae indicate that we perform a transition to an *imaginary proper time* τ. Both metrics, (2.1) and (2.8) are degenerate at $\tau = 0$, because $|\mathrm{det}g_{\mu\nu}(x)| = \tau^2 r^2$. The formal correspondence between these two metric forms is given by $\tau_M = -i\tau_E,\ \ \eta_M = i\eta_E$.

As we have discussed in the introduction, there is a remarkable correspondence between the quantum veto on any measurements at the intermediate stage of the exclusive process and the possibility to reduce the underlying theory to the path-integral formalism in Euclidean space. Being the solutions with the minimal action, the instantons also provide a local locking between the color and Euclidean spatial directions, thus making the color virtually invisible. An important ingredient of this structure is the $[O(4)]_{space}$ symmetry of an isotropic Euclidean space which can be mapped onto the $[SU(2)\times SU(2)]_{color}$ group of the color space. The presence of the spin connection explicitly provides the theory with the local invariance with respect to $[O(4)]_{space}$ rotations. The reader can easily check that the spin connection of the four-dimensional spherical coordinates exactly reproduces the pure-gauge asymptote of the BPST instanton [12]. The symmetry of the Euclidean space with the metric (2.8) does not allow for all six rotations of the $[O(4)]_{space}$ group, since the spin connection of this metric has only four (out of 12) non-vanishing components,

$$
{\omega_\eta}^{03} = -{\omega_\eta}^{30} = -1, \qquad {\omega_\phi}^{12} = -{\omega_\phi}^{21} = -1\ . \tag{2.11}
$$

Hence, the system acquires an "axis of quantization". Since the spin connection itself is the gauge field, it *can* be identified with the pure gauge of the Yang-Mills field. Then, in the (iso-)vector representation ${A_\mu}^{\alpha\beta}$ of the gauge field of the $O(4)$ group, we must have

$$
[{A_\mu}^{\alpha\beta}]_{pure\ gauge} = {\omega_\mu}^{\alpha\beta}\ , \tag{2.12}
$$

and the color space also acquires a quantization axis. The gauge fields of the $O(4)$ group have two projections on its two $SU(2)$-subgroups,

$$
({A^a}_\mu)_\pm = \frac{1}{4}\ \eta_\pm^{a\alpha\beta}{A_\mu}^{\alpha\beta} = \frac{1}{2}\big(\pm A_\mu^{0a} + \frac{1}{2}\epsilon^{a\alpha\beta}{A_\mu}^{\alpha\beta}\big)\ , \tag{2.13}
$$

where $\eta_\pm^{a\alpha\beta}$ are the 't Hooft symbols [13], and the subscripts ($\pm$) denote two chiral projections.[6] Since $\eta^{a03} = -\delta_{a3}$, and $\eta^{a12} = \delta_{a3}$, we have

$$
({A^3}_\eta)_\pm = \mp\frac{1}{2}\ {\omega_\eta}^{03} = \pm\frac{1}{2},\ \ ({A^3}_\phi)_\pm = \frac{1}{2}\ {\omega_\phi}^{12} = -\frac{1}{2}, \tag{2.14}
$$

which is compatible with the gauge condition $A^\tau = 0$ that we adopt for both the Euclidean and the Lorentz regimes of the process. One can easily find a representation for this potential which manifests its pure gauge origin,

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[6]The 't Hooft symbols carry indices $\alpha\beta$ of the local Cartesian coordinates, which in relativity theory correspond to the local inertial coordinates. This parallel is more straightforward that it might seem at the first glance. In the wedge dynamics, the local inertial observers move with respect to each other with velocities that depend on the observers' coordinates.

$$A_\mu(x) = (1/2)A_\mu^a(x)\sigma^a = S\partial_\mu S^{-1} \ . \tag{2.15}$$

Using the decomposition, $S = iu_0\mathbf{1} + u_a\sigma^a$, and $S^{-1} = -iu_0\mathbf{1} + u_a\sigma^a$ we arrive at

$$A_\mu(x) = (1/2)A_\mu^c(x)\sigma^c = -(\epsilon^{abc}u_a\partial_\mu u_b + u_0\partial_\mu u_c - u_c\partial_\mu u_0)\sigma^c \ . \tag{2.16}$$

By comparison with (2.14), and accounting for the unitarity, $SS^{-1} = 1$, we obtain a system of equations,

$$\begin{aligned}
-4(u_1\partial_\eta u_2 - u_2\partial_\eta u_1 + u_0\partial_\eta u_3 - u_3\partial_\eta u_0) &= \pm 1 \ , \\
-4(u_1\partial_\phi u_2 - u_2\partial_\phi u_1 + u_0\partial_\phi u_3 - u_3\partial_\phi u_0) &= -1 \ , \\
u_0^2 + u_a^2 &= 1 \ \ ,
\end{aligned} \tag{2.17}$$

which has a solution

$$\begin{aligned}
(u_0)_\pm = \mp 2^{-1/2}\cos\eta/2, \quad (u_3)_\pm = 2^{-1/2}\sin\eta/2 \ , \\
(u_1)_\pm = 2^{-1/2}\cos\phi/2, \quad (u_2)_\pm = 2^{-1/2}\sin\phi/2 \ .
\end{aligned} \tag{2.18}$$

Thus, we can conjecture that there exists a non-trivial solution of the Yang-Mills equations which has a pure gauge asymptote

$$\begin{aligned}
({A^3}_\eta)_\pm \to \ iS\partial_\eta S^{-1} = \pm\frac{1}{2}, \\
({A^3}_\phi)_\pm \to \ iS\partial_\phi S^{-1} = -\frac{1}{2}.
\end{aligned} \tag{2.19}$$

This asymptote has only one component in color space. The $\pm$ signs correspond to the right- and and left-handed projections on the $SU(2) \times SU(2)$ group. The components $({A^3}_\phi)_\pm$ are the same for both projections, and can be gauged out by one (Abelian) gauge transformation. However, he components $({A^3}_\eta)_\pm$ cannot. Furthermore, in the wedge dynamics, the field A_η is subjected to a non-trivial boundary condition that completely fixes the gauge $A^\tau = 0$, while the component A_ϕ is free of this kind of limitation.

III. SELF-DUAL SOLUTIONS IN THE WEDGE DYNAMICS.

In this section, we find the solutions of Euclidean QED which are compatible with the geometric background and boundary conditions of the wedge dynamics. We need the solutions that provide the minimum of Euclidean action. This minimum is well known to be reached on self-dual and anti-self-dual solutions of the Euclidean Yang-Mills equations. The condition for the self-duality (anti-self-duality) of the field tensor $F_{\mu\nu}$ reads as

$$F^*_{\mu\lambda} \equiv \mathrm{g}_{\mu\nu}\mathrm{g}_{\lambda\sigma}\frac{\epsilon^{\nu\sigma\rho\kappa}}{2\sqrt{\mathrm{g}}}F_{\rho\kappa} = \pm F_{\mu\lambda} \ . \tag{3.1}$$

Note that the definition of the dual tensor is different from the familiar definition in flat space. This modification is obvious. Indeed, the co- and contravariant tensor components are even of different dimensions. We are looking for a self-dual solution, which delivers a true minimum to the Euclidean action and becomes a pure gauge at $\tau \to \infty$. This solution is supposed to approach the limiting value given by the spin connection (2.19) when $r \to 0$, and locally in the rapidity direction (i.e., where the tetrad vectors $({e^\alpha}_\mu)_E$ form a *local Cartesian basis*). Since the asymptote of the solution (after it is analytically continued to Minkowski space, see Sec. IV) has only one color component at the

Cauchy surface $\tau = const$, we shall look for a *mono-colored* self-dual solution.[7] Embedded into the $SU(3)$ color group, these solutions correspond to the gluon fields A^3_μ and A^8_μ, i.e., the so-called "white gluons". Since the Gell-Mann matrices t^3_{ij} and t^8_{ij} are diagonal, these gluons (one in each of the three $SU(2)$ subgroups of the $SU(3)$ color group) only differentiate quarks by their color, but they do not change the color of a quark.

Let us choose the gauge of the wedge dynamics, $A_\tau = 0$, and denote,

$$A^3_\phi = \Phi(\tau, r, \eta), \quad A^3_R = R(\tau, r, \eta), \quad A^3_\eta = N(\tau, r, \eta) \ .$$

Since the field has only one color component, the commutator in the definition, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - [A_\mu, A_\nu]$, vanishes and the components of the field tensor are the same as in the Abelian case where

$$F_{\tau\eta} = \partial_\tau N, \quad F_{\tau\phi} = \partial_\tau \Phi, \quad F_{\tau r} = \partial_\tau R \ ,$$
$$F_{r\eta} = \partial_r N - \partial_\eta R, \quad F_{r\phi} = \partial_r \Phi, \quad F_{\eta\phi} = \partial_\eta \Phi \ . \tag{3.2}$$

The requirement of self-duality of the field (3.1) yields a system of equations,

$$\frac{\partial N}{\partial \tau} = \frac{\tau}{r} \frac{\partial \Phi}{\partial r} \ , \tag{3.3}$$

$$\frac{\partial \Phi}{\partial \tau} = -\ \frac{r}{\tau} \left[\frac{\partial N}{\partial r} - \frac{\partial R}{\partial \eta} \right] \ , \tag{3.4}$$

$$\frac{\partial R}{\partial \tau} = -\frac{1}{\tau r} \frac{\partial \Phi}{\partial \eta} \ . \tag{3.5}$$

The conditions of self-consistency for this system obviously coincide with the Yang-Mills equations. For example, one of such conditions is $\partial_\tau \partial_\eta R = \partial_\eta \partial_\tau R$. Using Eqs. (3.4) and (3.5), and excluding N with the aid of Eq. (3.3), we arrive at

$$\frac{\partial^2 \Phi}{\partial \tau^2} + \frac{1}{\tau} \frac{\partial \Phi}{\partial \tau} + \frac{1}{\tau^2} \frac{\partial^2 \Phi}{\partial \eta^2} = -\left[\frac{\partial^2 \Phi}{\partial r^2} - \frac{1}{r} \frac{\partial \Phi}{\partial r} \right] \ . \tag{3.6}$$

This equation is easily solved by separation of variables. The solution is readily found in the form,

$$\Phi(\tau, r, \eta) = c \ \lambda r \ J_1(\lambda r) \ K_{-i\nu}(\lambda \tau) e^{-\nu \eta} \tag{3.7}$$

In order to find similar equations for R and N, we have to use the self-consistency conditions of the third order. To derive them, we must begin with three equations of the second order, $\partial_\tau \partial_r \Phi = \partial_r \partial_\tau \Phi, \quad \partial_\eta \partial_r \Phi = \partial_r \partial_\eta \Phi, \quad \partial_\eta \partial_\tau \Phi = \partial_\tau \partial_\eta \Phi$, which read as

$$\frac{\partial}{\partial \tau} \left(\frac{r}{\tau} \frac{\partial N}{\partial \tau} \right) = \frac{\partial}{\partial r} \left(\frac{r}{\tau} \frac{\partial R}{\partial \eta} - \frac{r}{\tau} \frac{\partial N}{\partial r} \right) \ , \tag{3.8}$$

$$\frac{\partial}{\partial \eta} \left(\frac{r}{\tau} \frac{\partial N}{\partial \tau} \right) = -\frac{\partial}{\partial r} \left(\tau r \frac{\partial R}{\partial \tau} \right) \ , \tag{3.9}$$

---

[7] Such a solution will indeed be found. Its stability with respect to the interaction with the fermion modes or other gluon modes is a separate issue.

$$\frac{\partial}{\partial \eta}\left(\frac{r}{\tau}\frac{\partial N}{\partial r} - \frac{r}{\tau}\frac{\partial R}{\partial \eta}\right) = \frac{\partial}{\partial \tau}\left(\tau r \frac{\partial R}{\partial \tau}\right) . \tag{3.10}$$

Excluding N from Eqs. (3.9) and (3.10), we derive an equation for R,

$$\left[\frac{\partial^2}{\partial \tau^2} + \frac{1}{\tau}\frac{\partial}{\partial \tau} + \frac{1}{\tau^2}\frac{\partial^2}{\partial_\eta^2}\right]\left(\tau\frac{\partial R}{\partial \tau}\right) = -\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{1}{r^2}\right]\left(\tau\frac{\partial R}{\partial \tau}\right) , \tag{3.11}$$

with the solution,

$$\tau\frac{\partial R}{\partial \tau} = c_1 \ \lambda_1 J_1(\lambda_1 r) K_{-i\nu_1}(\lambda_1 \tau) e^{-\nu_1 \eta} . \tag{3.12}$$

In the same way, excluding R from Eqs. (3.8) and (3.9), we derive an equation for N,

$$\left[\frac{\partial^2}{\partial \tau^2} + \frac{1}{\tau}\frac{\partial}{\partial \tau} + \frac{1}{\tau^2}\frac{\partial^2}{\partial_\eta^2}\right]\left(\frac{1}{\tau}\frac{\partial N}{\partial \tau}\right) = -\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right]\left(\frac{1}{\tau}\frac{\partial N}{\partial \tau}\right) , \tag{3.13}$$

which has the solution,

$$\frac{1}{\tau}\ \frac{\partial N}{\partial \tau} = c_2 \ J_0(\lambda_2 r) K_{-i\nu_2}(\lambda_2 \tau) e^{-\nu_2 \eta} . \tag{3.14}$$

Substituting the solutions (3.7), (3.12), and (3.14) into the original system (3.3)-(3.5), we find the following relations among the constants,

$$\nu_1 = \nu_2 = \nu, \quad \lambda_1 = \lambda_2 = \lambda, \quad c_2 = \pm c, \quad c_1 = \mp \nu c .$$

The second solutions of ordinary differential equations that we solve after the separation of variables are dropped for various reasons: The Bessel function $I_{-i\nu}(\lambda\tau)$ explodes at large τ. The Bessel functions $Y_0(\lambda r)$ and $Y_1(\lambda r)$ are singular at the origin. The second exponent, $e^{+\nu\eta}$ is accounted for, since $K_\nu(x) = K_{-\nu}(x)$. Finally, we may write the solution as

$$\begin{aligned} \Phi_{\nu,\lambda,\vec{r}_0}(x) &= c\lambda r J_1(\lambda r) K_{-i\nu}(\lambda\tau) e^{-\nu\eta} , \\ R_{\nu,\lambda,\vec{r}_0}(x) &= \mp c \ \nu\lambda J_1(\lambda r) M_{-1,-i\nu}(\lambda\tau) e^{-\nu\eta} , \\ N_{\nu,\lambda,\vec{r}_0}(x) &= \pm c \ J_0(\lambda r) M_{1,-i\nu}(\lambda\tau) e^{-\nu\eta} , \end{aligned} \tag{3.15}$$

where $r = |\vec{r} - \vec{r}_0|$, and $\vec{r}_0$ labels the position of the center in the transverse xy-plane. The new functions,

$$M_{m,-i\nu}(\tau) = \int_\infty^\tau K_{-i\nu}(t)\ t^m \ dt, \tag{3.16}$$

are similar to the functions $R^{(j)}_{m,-i\nu}(\tau) = \int H^{(j)}_{-i\nu}(t)\ t^m \ dt$, used extensively in paper [III]. A particular choice of the infinite lower limit in Eq. (3.16) leads to the solutions that vanish at $\tau \to \infty$. Another form of the solution, in which the variables are not separated explicitly, but are extremely useful for the future analysis, is obtained by means of the (two-sided) Laplace transform, e.g.,

$$\Phi_{\theta,\lambda,\vec{r}_0}(x) = \int_{-\infty}^{\infty} e^{\nu\eta}\Phi_{\nu,\lambda,\vec{r}_0}(x) d\nu , \tag{3.17}$$

In this way, we trade the "quantum number" ν for the quantum number θ which is the Euclidean analog of the Minkowski rapidity. Computing the Laplace integral, we use the formula,

$$\int_{-\infty}^{\infty} e^{\nu(\theta-\eta)} \ K_{i\nu}(\lambda\tau)\ d\nu = 2\int_0^{\infty} \cosh[\nu(\eta-\theta)] \ K_{i\nu}(\lambda\tau)\ d\nu = \pi e^{-\lambda\tau\cos(\eta-\theta)} , \tag{3.18}$$

where the integral is convergent at $|\mathrm{Re}(\eta-\theta)|\leq\pi/2$, and can be analytically continued. The solution with a given Euclidean rapidity θ reads as

$$\Phi_{\theta,\lambda,\vec{r}_0}(x_E) = c\ \lambda r J_1(\lambda r) e^{-\lambda\tau_E\cos(\eta_E-\theta_E)}\ , \tag{3.19}$$

$$\begin{aligned} R_{\theta,\lambda,\vec{r}_0}(x_E) &= \mp c\ \lambda J_1(\lambda r)\tan(\eta_E-\theta_E) e^{-\lambda\tau_E\cos(\eta_E-\theta_E)}\ , \\ N_{\lambda,\vec{r}_0}(x_E) &= \mp c\ J_0(\lambda r)\ \frac{1+\lambda\tau_E\cos(\eta_E-\theta_E)}{\cos^2(\eta_E-\theta_E)} e^{-\lambda\tau_E\cos(\eta_E-\theta_E)}\ , \end{aligned} \tag{3.20}$$

where we have explicitly labeled the Euclidean variables. The condition $|\eta_E-\theta_E|\leq\pi/2$ means that these solutions occupy only a quarter of the Euclidean $t_E z_E$-plane. Indeed, in order to satisfy this condition, we must have $|\eta_E|\leq\pi/4$ and $|\theta_E|\leq\pi/4$, satisfied separately. Introducing the Euclidean energy $\omega_E=\lambda\cos\theta$ and the Euclidean longitudinal momentum $p^z_E=\lambda\sin\theta$, along with $t_E=\tau_E\cos\eta_E$ and $z_E=\tau_E\sin\eta_E$, we may rewrite the exponent in Eqs. (3.20) as

$$e^{-\lambda\tau_E\cos(\eta_E-\theta_E)} = e^{-(\omega_E t_E+p^z_E z_E)},$$

which clearly indicates that our solutions are inhomogeneous waves adjacent to the hypersurface $\tau^2_E=t^2_E+z^2_E=0$. Another form of the solution can be obtained by means of the (Abelian) gauge transform, i.e., by adding the gradient,

$$A'_\mu(x_E) = A_\mu(x_E) - \partial_\mu\chi(x_E),$$

of the function

$$\chi(x_E) = \mp c\ \tan(\eta_E-\theta_E) J_0(\lambda r)\ . \tag{3.21}$$

This transform does not modify $A_\phi=\Phi$, and it results in the field that satisfies the boundary condition, $A_\eta(\tau_E=0)=0$,

$$\begin{aligned} R'_{\theta,\lambda,\vec{r}_0}(x_E) &= \mp c\lambda J_1(\lambda r)\tan(\eta_E-\theta_E)[e^{-\lambda\tau_E\cos(\eta_E-\theta_E)}-1]\ , \\ N'_{\lambda,\vec{r}_0}(x_E) &= \mp c J_0(\lambda r)\left[\frac{e^{-\lambda\tau_E\cos(\eta_E-\theta_E)}-1}{\cos^2(\eta_E-\theta_E)} + \lambda\tau_E\frac{e^{-\lambda\tau_E\cos(\eta_E-\theta_E)}}{\cos(\eta_E-\theta_E)}\right]\ . \end{aligned} \tag{3.22}$$

When $\tau\to\infty$, the solution with components (3.19) and (3.22) becomes a pure gauge; then it is just the gradient of the function (3.21). Furthermore, along a singular line $r=0$ and at $\eta_E=\theta_E$ this pure gauge potential approaches its asymptotic value which coincides with the projection of the $O(4)$ spin connection onto the $SU(2)$ potentials. Since at large τ, the states of the wedge dynamics are highly localized in the rapidity direction around the rapidity θ, it is quite natural that the limit of the spin connection is reached exactly at $\eta=\theta$. In order to comply with the asymptotic condition (2.19), we have to take $c=1/2$, though in general, we have the linear equations which leave the common normalization factor not defined.

An important remark should be made at this point. *The pure-gauge asymptotic behavior of this solution is enforced by the boundary condition imposed on the potential A_η at $\tau=0$.* This is a physical requirement of continuity brought from the physical Minkowski world.

The pure-gauge asymptote of Eq. (3.22) becomes singular at the two isolated points, $\theta_E=\pi/4$, $\eta_E=-\pi/4$, and $\theta_E=-\pi/4$, $\eta_E=\pi/4$, which correspond to the limit of the infinite momentum frame (or the null-plane dynamics). In the same limit the solution (3.20) (which otherwise tends to zero at $\tau\to\infty$), is also singular. This solution does not obey the boundary conditions that provide a complete fixing of the gauge $A^\tau=0$. The solution (3.22) is regular in all these limits.[8] For large values of the parameter λ, which can be put in correspondence with the transverse momentum p_t of the plane-wave solutions in Minkowski space, the Euclidean life-time of the solution given by Eqs. (3.19) and (3.22) is very short.[9] It has a noticeable amplitude only in the vicinity of $\tau_E = 0$ and thus, it very much resembles surface (Tamm) states in condensed matter physics. In the next section we show, that the ephemeron solutions can be analytically continued to Minkowski space, where they immediately become the asymptotic states, similar to those truncated by the LSZ reduction procedure in the standard scattering problem.

IV. PROPAGATING STATES OF WEDGE DYNAMICS.

In this section we show that the ephemeron solutions in Euclidean space can be obtained by the analytic continuation of the propagating waves of the wedge dynamics. This continuation involves not only the proper time, but the rapidity variables also.[10] In the framework of wedge dynamics with the gauge $A_\tau = 0$, the solutions of the linearized Yang-Mills equations (thus, with a given color, which is not indicated) were found in paper [III]. The two modes of the radiation field with components $A_m = (A_x, A_y, A_\eta)$ are

$$V^{(TE)}_{\vec{k},\nu}(x) = \frac{e^{-\pi\nu/2}}{2^{5/2}\pi k_t} \begin{pmatrix} k_y \\ -k_x \\ 0 \end{pmatrix} H^{(2)}_{-i\nu}(k_t\tau) e^{i\nu\eta + i\vec{k}\vec{r}} \ ,$$

$$V^{(TM)}_{\vec{k},\nu}(x) = \frac{e^{-\pi\nu/2}}{2^{5/2}\pi k_t} \begin{pmatrix} \nu k_x R^{(2)}_{-1,-i\nu}(k_t\tau) \\ \nu k_y R^{(2)}_{-1,-i\nu}(k_t\tau) \\ -R^{(2)}_{1,-i\nu}(k_t\tau) \end{pmatrix} e^{i\nu\eta + i\vec{k}\vec{r}} \ . \tag{4.1}$$

The mode $V^{(TM)}$ is constructed from the functions (see paper [III]) $R^{(2)}_{m,-i\nu}(x) = \int H^{(2)}_{-i\nu}(t) t^m dt$ (originally defined as the indefinite integrals) corresponding to the boundary condition of vanishing gauge field at $\tau = 0$. This guarantees continuous behavior of the field at $\tau = 0$. Indeed, as $\tau \to 0$, the normal and the tangential directions become degenerate. As long as $A^\tau = 0$ is the gauge condition, continuity requires that $A^\eta \to 0$ as $\tau \to 0$. The transverse electric mode $V^{(TE)}$ and transverse magnetic mode $V^{(TM)}$ have quantum numbers of boost ν and of transverse momentum $\vec{k} = (k_x, k_y)$. Let us trade the latter for the position $\vec{r}_0$ of the azimuthally symmetric mode $V_{\nu,\lambda\vec{r}_0}$, acting as follows. We introduce

$$V^m_{\vec{r}_0,\nu}(x) = \int \frac{d^2\vec{k}}{2\pi} \, V^m_{\vec{k},\nu}(x) \, e^{-i\vec{k}\vec{r}_0} \ , \tag{4.2}$$

and integrate only over the angle between the vectors $\vec{k}$ and $\vec{r} - \vec{r}_0$, leaving the dk_t integral not integrated, and denoting (for convenience) k_t as λ. Furthermore, let us write the answer in terms of the radial and azimuthal components A_r and A_ϕ,

$$A_r = A^r = \cos\phi \ A^x + \sin\phi \ A^y, \quad A_\phi = r^2 A^\phi = -r\sin\phi \ A^x + r\cos\phi \ A^y \ . \tag{4.3}$$

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[8]These limits are sensitive to the order in which they are applied. This ambiguity can be traced back to the Minkowski space, where the normal and tangent direction on the hypersurface of the constant $\tau \to 0$ are degenerate. The way one should resolve this ambiguity depends on the observable under consideration.

[9]We would suggest to call this object *ephemeron* in order not to create an image of a particle and emphasize the ephemeral nature of this field configuration.

[10]Recently, Shuryak and Zahed used the analytic continuation to Euclidean rapidity in their attempt to estimate the effect of instantons on the high-energy exclusive scattering between two *a priori* factorized partons which are treated in the eikonal approximation [14].

The transverse electric mode $V^{(TE)}$ has only a ϕ-component,

$$\Phi_{\nu,\lambda,\vec{r}_0}(x_M) = -2^{-5/2}\pi^{-1}\lambda r J_1(\lambda r) e^{-\pi\nu/2} H^{(2)}_{-i\nu}(\lambda\tau) e^{i\nu\eta} \ , \tag{4.4}$$

The transverse magnetic mode $V^{(TM)}$ has only r- and η-components,

$$R_{\nu,\lambda,\vec{r}_0}(x_M) = 2^{-5/2}\pi^{-1}\nu\lambda J_1(\lambda r) e^{-\pi\nu/2} R^{(2)}_{-1,-i\nu}(\lambda\tau) e^{i\nu\eta} \ , \tag{4.5}$$

$$N_{\nu,\lambda,\vec{r}_0}(x_M) = -2^{-5/2}\pi^{-1} i J_0(\lambda r) e^{-\pi\nu/2} R^{(2)}_{1,-i\nu}(\lambda\tau) e^{i\nu\eta} \ , \tag{4.6}$$

where, as previously, $r = |\vec{r} - \vec{r}_0|$. Now, one can immediately establish the connection between the components (4.4)-(4.6) of the radiation field in Minkowski space and the components (3.15) of the self-dual solution in Euclidean space. Since

$$-\frac{i\pi}{2}\ e^{-\pi\nu/2} H^{(2)}_{-i\nu}(\lambda\tau_M) = -\frac{i\pi}{2}\ e^{-\pi\nu/2} H^{(2)}_{-i\nu}(\lambda\tau_E e^{-i\pi/2}) = K_{-i\nu}(\lambda\tau_E) \ , \tag{4.7}$$

we may guess that the desired prescription is,

$$\tau_M = e^{-i\pi/2}\tau_E \ , \eta_M = e^{i\pi/2}\eta_E \ .$$

Since the function $K_{i\nu}(z)$ has a branching point at the origin, this conjecture is not yet completely certain. The issue can be clarified only after we get the Minkowski modes with the quantum number of rapidity. These modes can be obtained by means of the Fourier transform,

$$V_{\theta,\lambda,\vec{r}_0}(x) = \int_{-\infty}^{+\infty} \frac{d\nu}{(2\pi)^{1/2}} e^{-i\nu\theta} V_{\nu,\lambda,\vec{r}_0}(x) \ . \tag{4.8}$$

Using the following integral representation for the Hankel functions,

$$e^{-\pi\nu/2} e^{i\nu\eta} H^{(2)}_{-i\nu}(\lambda\tau) = \frac{i}{\pi}\int_{-\infty}^{\infty} e^{-i\lambda\tau\cosh(\theta-\eta)} e^{i\nu\theta} d\theta \ , \tag{4.9}$$

and changing the order of integration in the expressions (4.5) and (4.6) for the A_r and A_η components, we arrive at

$$\Phi_{\theta,\lambda,\vec{r}_0}(x_M) = \frac{1}{4\pi^{3/2}} \lambda r J_1(\lambda r) e^{-i\lambda\tau_M \cosh(\theta_M - \eta_M)} \ , \tag{4.10}$$

$$R'_{\theta,\lambda,\vec{r}_0}(x_M) = \frac{i}{4\pi^{3/2}} \lambda J_1(\lambda r) \tanh(\eta_M - \theta_M)\big[e^{-i\lambda\tau_M\cosh(\theta_M-\eta_M)} - 1\big] \ , \tag{4.11}$$

$$N'_{\theta,\lambda,\vec{r}_0}(x_M) = -\frac{i}{4\pi^{3/2}} J_0(\lambda r)\left[\frac{e^{-i\lambda\tau_M\cosh(\theta_M-\eta_M)} - 1}{\cosh^2(\theta_M-\eta_M)} + i\lambda\tau_M \frac{e^{-i\lambda\tau_M\cosh(\theta_M-\eta_M)}}{\cosh(\theta_M-\eta_M)}\right] \ . \tag{4.12}$$

This representation clearly satisfies the boundary condition, $A_\eta(\tau_M = 0) = 0$. As it has been discussed in paper [III], this boundary condition fixes the gauge $A^\tau = 0$ completely. The solution that does not satisfy this boundary condition, but can be "analytically continued" to the solution (3.22) in Euclidean space can be obtained by means of the gauge transform,

$$A_\mu(x_M) = A'_\mu(x_M) + \partial_\mu \chi(x_M),$$

with the function

$$\chi(x_M) = i\ \tanh(\eta_M - \theta_M) J_0(\lambda r)\ , \tag{4.13}$$

and is as follows,

$$R_{\theta,\lambda,\vec{r}_0}(x_M) = \frac{i}{4\pi^{3/2}} \lambda J_1(\lambda r) \tanh(\eta_M - \theta_M) e^{-i\lambda\tau_M \cosh(\theta_M - \eta_M)}\ , \tag{4.14}$$

$$N_{\nu,\lambda,\vec{r}_0}(x_M) = -\frac{i}{4\pi^{3/2}} J_0(\lambda r) \left[\frac{e^{-i\lambda\tau_M \cosh(\theta_M - \eta_M)}}{\cosh^2(\theta_M - \eta_M)} + i\lambda\tau_M \frac{e^{-i\lambda\tau_M \cosh(\theta_M - \eta_M)}}{\cosh(\theta_M - \eta_M)} \right]\ . \tag{4.15}$$

Comparing Eqs. (4.10)-(4.12) with the expressions (3.19) and (3.22) of the Euclidean solutions, we can formulate the complete set of substitutions that are required to pass over from the Minkowski solutions, to the Euclidean ones,

$$\begin{aligned} \tau_M \to -i\tau_E, \quad \eta_M \to i\eta_E, \quad \theta_M \to i\theta_E\ , \\ |\eta_E| \le \pi/4\ , \quad |\theta_E| \le \pi/4\ , \\ A_\phi(x_M) \to A_\phi(-i\tau_E, i\eta_E) = A_\phi(x_E)\ , \\ A_r(x_M) \to A_r(-i\tau_E, i\eta_E) = A_r(x_E)\ , \\ A_\eta(x_M) \to -i\ A_\eta(-i\tau_E, i\eta_E) = A_\eta(x_E)\ . \end{aligned} \tag{4.16}$$

On can easily see that the two signs of A_r and A_η of the Euclidean solutions correspond to the two linear combinations, $V^\pm = 2^{-1/2}(V^{(TE)} \pm V^{(TM)})$, which are the propagating waves with two circular polarizations[11]. The last one of these equations duplicates Eq.(2.10) and thus, is consistent with the view of the transition to the Euclidean metric as an analytic continuation of the time-like tetrad vector ${e_0}^\tau$. If we assume that the final states occupy the entire future domain of the collision, then this transition takes place on the hypersurface $\tau = 0$ where the metric of the wedge dynamics (both in Minkowski and Euclidean versions) is degenerate, i.e., $\det|g_{\mu\nu}| = 0$.

The analytic continuation of the tetrad vector ${e_0}^\tau$ can be employed to find the Euclidean version of the fermion wave function. In Minkowski space, the positive-frequency right-handed state is described by

$$\psi_R(p_t, \theta_M; \tau_M, \eta_M) = \frac{1}{4\pi^{3/2} p_t} \begin{pmatrix} e^{(\eta_M - \theta_M)/2}\ p_t \\ -e^{-(\eta_M - \theta_M)/2}\ (p_x + ip_y) \end{pmatrix} e^{-ip_t\tau_M \cosh(\eta_M - \theta_M)}\ . \tag{4.17}$$

After analytic continuation, this spinor becomes

$$\psi_R(p_t, \theta_E; \tau_E, \eta_E) = \frac{1}{4\pi^{3/2} p_t} \begin{pmatrix} e^{i(\eta_E - \theta_E)/2}\ p_t \\ -e^{-i(\eta_E - \theta_E)/2}\ (p_x + ip_y) \end{pmatrix} e^{-p_t\tau_E \cos(\eta_E - \theta_E)}\ , \tag{4.18}$$

which also corresponds to the inhomogeneous wave that flares up for the time $\sim 1/p_t$ near the edge ($\tau = 0$) of the Euclidean evolution.

V. EUCLIDEAN ACTION AND TOPOLOGICAL CHARGE.

As has been explained in the Introduction, we are looking for the fluctuations that can minimize the Euclidean action of the system of colliding nuclei along the path that ends at the moment of the nuclei intersection. One may hope that this optimal path will also include the ephemeron configurations which can be then continued to Minkowski

[11]The Minkowski solutions $V^\pm$ have an extra factor $(2\pi)^{-3/2}$ due to the physical normalization of these solutions as the one-particle states.

space as the propagating modes of the expanding quark-gluon system. Starting from the fields given by the Eqs. (3.2) we may find the Euclidean action of the ephemeron,

$$S_{\scriptscriptstyle E} = \frac{1}{4g^2}\int d^4x\sqrt{\mathrm{g}}\;\mathrm{g}^{\mu\rho}\mathrm{g}^{\nu\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}$$

$$= \frac{1}{g^2}\int d^4x\;\tau r\left[\frac{1}{\tau^2}\left(\frac{\partial\Phi}{\partial\tau}\right)^2 + \frac{1}{r^2}\left(\frac{\partial N}{\partial\tau}\right)^2 + \left(\frac{\partial R}{\partial\tau}\right)^2\right]. \tag{5.1}$$

In the same way, we compute the topological charge

$$Q = \frac{1}{32\pi^2}\int d^4x\sqrt{\mathrm{g}}\;\frac{\epsilon^{\mu\nu\rho\sigma}}{2\sqrt{\mathrm{g}}}\;F^a_{\mu\nu}F^a_{\rho\sigma} = \pm\,\frac{g^2}{8\pi^2}\;S_{\scriptscriptstyle E}\;. \tag{5.2}$$

Thus, we have a standard relation between the action and the winding number, well known for the instanton field configurations, which is true for any self-dual Euclidean field. Using the expressions (3.19) and (3.22) for the field components, we arrive at

$$S_{\scriptscriptstyle E} = \frac{c^2\lambda^4}{g^2}\int_0^\infty \tau d\tau\int_0^{R_t} rdr\int_0^{2\pi} d\phi\int_{-\pi/4}^{\pi/4} d\eta\; e^{-2\lambda\tau\cos(\eta-\theta)}\left[J_0^2(\lambda r)+J_1^2(\lambda r)\right]\;, \tag{5.3}$$

where the upper limit R_t in the integral over the transverse radius r is introduced in order to isolate the divergence naturally inherent in the action of the free infinitely propagating field. The value of this parameter is limited from above by the transverse area of the nuclei intersection. Practically, the upper limit is much lower, since the ephemeron with large λ can be resolved only by a process with the large transverse momentum transfer. Thus, R_t should be of the order of λ^{-1}. All integrations in Eq. (5.3) can be carried out analytically and yield

$$S_{\scriptscriptstyle E} = \frac{c^2\pi\lambda^2R_t^2}{g^2}\;\frac{1}{\cos 2\theta_{\scriptscriptstyle E}}\left[J_0^2(\lambda R_t)+J_1^2(\lambda R_t) - \frac{J_0(\lambda R_t)J_1(\lambda R_t)}{\lambda R_t}\right]. \tag{5.4}$$

Even at finite R_t, this action diverges at $\theta\to\pm\pi/4$, i.e., when the Minkowski rapidity of the particle tends to infinity and thus it infinitely moves freely (in Minkowski space). When $\lambda R_t > 1$, the asymptotic expansion of the Bessel functions is sufficiently accurate. Then

$$S_{\scriptscriptstyle E} \approx \frac{2c^2\lambda R_t}{g^2\cos 2\theta_{\scriptscriptstyle E}}\left[1 - \frac{\sin(2\lambda R_t)}{2\lambda R_t}\right]. \tag{5.5}$$

In order to understand the physical mechanism that provides this Euclidean action, let us use the representation of topological charge via the divergence of the Chern-Simons current,

$$Q = \frac{1}{4\pi^2}\oint d\sigma_\mu K^\mu = \frac{1}{4\pi^2}\int d^4x\;\partial_\mu K^\mu, \tag{5.6}$$

where

$$K^\mu = \frac{1}{4}\epsilon^{\mu\nu\rho\sigma}\left[A^a_\nu\partial_\rho A^a_\sigma + \frac{1}{3}\epsilon_{abc}A^a_\nu A^b_\rho A^c_\sigma\right]. \tag{5.7}$$

The second term (usually the major one) identically vanishes since the ephemeron field has only one color component. It is easy to show, that the components of K^μ can be presented in a compact form,

$$K^\tau = \pm\frac{1}{4}\sqrt{\mathrm{g}}\;\mathrm{g}^{lm}A_lE_m\;,\quad K^l = -\frac{1}{4}\epsilon^{lmn}A_mE_n = \frac{1}{4}[\mathbf{E}\times\mathbf{A}]^l\;, \tag{5.8}$$

where $\mathrm{g}^{lm} = \mathrm{diag}[1, r^{-2}, \tau^{-2}]$, $E_m = -\partial_\tau A_m$ is the electric field strength, and ϵ^{lmn} is the three-dimensional Levi-Civita tensor, $\epsilon^{r\phi\eta} = 1$.[12] We can see now, that the spatial components of the topological current coincide with the

[12]Deriving the expression for K^τ, we explicitly used the condition of self-duality; this is where the alternating sign came from. The alternating sign in the spatial components will come from the fields defined by Eq.(3.22).

canonical vector of the photon spin, thus being related to the polarization properties of the ephemeron field. Using the representation (5.7), we can write down the topological charge as a sum of the three surface integrals,

$$
\begin{aligned}
Q = \frac{1}{2\pi}\Big\{ \int_{-\pi/4}^{\pi/4} d\eta \int_0^{R_t} dr \big[K^\tau(\infty,r,\eta) - K^\tau(0,r,\eta)\big] \\
+ \int_{-\pi/4}^{\pi/4} d\eta \int_0^{\infty} d\tau \big[K^r(\tau,R_t,\eta) - K^r(\tau,0,\eta)\big] \\
+ \int_0^{\infty} d\tau \int_0^{R_t} dr \big[K^\eta(\tau,r,\pi/4) - K^\eta(\tau,r,-\pi/4)\big] \Big\} \ .
\end{aligned}
\tag{5.9}
$$

Next, it is easy to show, that $K^\tau(\infty,r,\eta)=0$, $K^\tau(0,r,\eta)=0$, and $K^r(\tau,0,\eta)=0$; this immediately follows from Eq. (5.8) and the expressions for the field components. Furthermore, for a large λR_t, the contribution of $K^r(\tau,R_t,\eta)$ is suppressed by an extra factor $(\lambda R_t)^{-1}$. As a result, Q is defined mainly by the last line in Eq. (5.9). Thus, the topological charge and, by virtue of (5.2), the Euclidean action are defined by the difference of the flux of the rapidity component K^η of the canonical spin between the hyper-planes $\eta_{_E}=\pm\pi/4$ ($\eta_{_M}=\pm\infty$),

$$
\begin{aligned}
Q &\approx \frac{c^2\lambda^2}{8\pi}\int_0^\infty d\tau \int_0^{R_t} dr \Big[\lambda r\ J_1^2(\lambda r)\ \sin(\eta-\theta)\ e^{-\lambda\tau\cos(\eta-\theta)}\Big]_{\eta=-\pi/4}^{\eta=\pi/4} \\
&= \frac{c^2\lambda^2 R_t^2}{8\pi}\ \frac{1}{\cos 2\theta_{_E}}\ \Big[J_0^2(\lambda R_t) + J_1^2(\lambda R_t) - 2\frac{J_0(\lambda R_t)J_1(\lambda R_t)}{\lambda R_t}\Big] \ .
\end{aligned}
\tag{5.10}
$$

This effect is clearly of a topological nature. Indeed, by observation, the radial component (3.22)

$$
A_r = R'_{\theta,\lambda,\vec{r}_0}(x_{_E}) \propto \tan(\eta_{_E}-\theta_{_E}) \ ,
$$

of the Euclidean field, or alternatively, the the radial component (4.11),

$$
A_r = R'_{\theta,\lambda,\vec{r}_0}(x_{_M}) \propto \tanh(\eta_{_M}-\theta_{_M}) \ ,
$$

of the Minkowski field change their sign when the coordinate η crosses the rapidity center θ of the state. The function that describes this transition is shaped exactly as a classical kink. Since the component A_ϕ does not changes its sign, the observer that moves along the η direction will encounter a gradual change of the circular polarization (direction of the photon spin) of the ephemeron field.[13] This is also seen directly from the spin density K^η (the integrand in Eq. (5.10)). Thus, we encounter a new example of Thomas precession, inherent in all states with spin in the wedge dynamics. For the Dirac field, this effect has been discussed in paper [II]. In both cases, the effect is due to the *curvature* of the hypersurface of constant proper time τ, along which the states of the wedge dynamics are normalized by the system of the moving observers. Indeed, in curvilinear coordinates, the spin connection

$$
\omega_\eta^{\tau\eta} = e_\alpha^{\ \tau} e_\beta^{\ \eta}\omega_\eta^{\ \alpha\beta} = \frac{1}{\tau}\omega_\eta^{\ 03} \ ,
$$

is proportional to the curvature τ^{-1}; it is large at small τ, and vanishes at large τ. This means that at large τ, the local physics remains unaffected by the curvature.

---

[13] One can easily recognize the similarity between the polarization properties of the ephemeron field and those of the system of two photons with opposite spins and moving in opposite directions. Such a system emerges in the decay $\pi^0 \to 2\gamma$. In that case, the amplitude of the process includes an Abelian axial anomaly which is proportional to the topological charge, though the field of the photons is not self-dual, and the topological charge is not proportional to the minimal action.

Finally, solely for pedagogical reasons, we shall try to connect the existence of the non-vanishing (and not integer) topological charge Q with some kind of tunneling. The first thing to do is to find a "potential barrier". In our case, the barrier emerges because in the wedge dynamics any state is detected by a system of local observers moving with different velocities. Therefore, a transfer along the surface of constant τ is impossible without acceleration.[14] The physical effect of this acceleration is Thomas precession, which rotates the spin of the state in opposite directions when the state is detected on different sides of the rapidity center of the wave packet. It is exactly the work done by the forces of inertia which creates a potential barrier between these two degenerate states. One can say, that this degeneracy is provided by the tunneling.

VI. CONCLUSION

The theory of ultrarelativistic nuclear collisions is a challenging field of research where the price of correctly asked questions amounts to the value of the practical results. From the perspective of the work accomplished in the previous [1]- [6] and the present paper, we may claim that the scenario of ultrarelativistic nuclear collisions, despite an enormous complexity of the phenomenon, can be at least drafted from first principles. Its design must include two key ingredients, dynamical definition of the "final" states as the collective modes of the quark-gluon matter at the intermediate stage of the evolution, and the mechanism responsible for the breakdown of the nuclei coherence. The existence of the Euclidean ephemeron solutions seems to resolve the issue of the nuclei decoherence exactly in the same way as the instantons solve the problem of their integrity before the collisions. A Euclidean path through the instanton liquid provides the least action to a confined group of quarks and make the color degrees of freedom invisible. The coherence of the nuclear wave function is lost because the path of the least Euclidean action includes new field configurations which are the Euclidean images of the propagating states and do not interlock the color and space directions. At the last instance before the collision the nuclei just *become unbound*.

The physical meaning of ephemerons is exactly the same as the meaning of instantons. Indeed, one can easily check that

(i) Since the Euclidean field strength tensor $F^{(E)}_{\mu\nu}$ is self-dual, the Euclidean energy-momentum tensor vanishes, $T^{(E)}_{\mu\nu}(F^{(E)}) \equiv 0$;

(ii) The analytic continuation $x_E \to x_M$ leads to the $F^{(M)}_{\mu\nu}$, in which $B^k_E \to B^k_M$, and $E^k_E \to iE^k_M$;

(iii) The energy-momentum tensor of the Minkowski metric computed on these analytically continued fields vanishes also, $T^{(M)}_{\mu\nu}(F^{(M)}) \equiv 0$.

This is a clear indication that the ephemerons indeed are the pure vacuum fluctuations carrying no energy or momentum. In the Euclidean version of the wedge dynamics, they must be included into the ensemble of the classical field configurations on the same footing as instantons. A distinctive feature of the wedge dynamics is the localization of the states with a given velocity. Therefore, at large τ, the wave packets representing the two nuclei are well separated, and each of them can be studied locally in its own rest frame. This property does not change in the Euclidean wedge dynamics, which provides *a unique opportunity to consider both the confining regime and the interaction of the two nuclei in a common Euclidean projection.* If no interaction between the nuclei occurs, then the ephemerons appear

[14] The reader should not be confused by this "motion in the space-like direction". In Euclidean space, all directions are space-like. Instantons also do not necessarily provide tunneling between the configurations which are separated by a time interval.

and vanish as the pure gauge fields without any material trace. Only a real interaction, that explicitly breaks the translational symmetry, can supply them with the energy and excite them as the propagating fields.

The instanton physics, despite the global nature of mathematical theorems related to the topological field configurations, is, as a matter of fact, the *local physics* in isotropic space. Therefore, we may expect, that at $\tau \geq 1fm$ all physical mechanisms that stabilize the instanton liquid [7] are not corrupted, and the instantons of the size $\rho_i < \tau$ do not have real competitors. The ephemerons with small λ (i.e. large size $\rho_e = \lambda^{-1}$), despite the fact that their fields are weak, are not likely to survive, e.g., the long lasting interactions with light quarks. Only the short-lived ephemerons of small size are expected to be truly active, since they show up only at the latest moments before the nuclei intersect. Then, they have many properties that one may wish to associate with partons. One may also see sufficient differences. They cannot be factorized out of the fast-moving nuclei as the point-like color charges or the independent plane waves. At the earliest moments, they are widely extended in the rapidity direction and localized in the transverse plane. They are "made of" the electric and magnetic fields with highly non-trivial polarization. They are likely to interact by chromo-magnetic rather than by chromo-electric forces. They are the fluctuations that can be resolved in the course of a collision and become free (with all reservations regarding the final-state interactions) only after the collision. It is still necessary to learn how the ephemerons interact with quarks and between each other, and if there exists a critical density of ephemerons in the transverse plane.

Regardless of the ephemeron's size, its Euclidean action rapidly grows when $|\theta_E| \to \pi/4$, $|\tan\theta_E| \to 1$. Then,

$$e^{-S_E} \propto \exp\left\{ -\frac{\lambda R_t}{g^2 \cos 2\theta_E} \right\} .$$

The weight $\exp[-S_E]$ strongly suppresses the Euclidean fluctuations with the extreme rapidities θ_E. After analytic continuation, these fluctuations would become the partons with the largest Minkowski rapidities θ_M which, it their turn, correspond to the smallest $x_F \sim e^{-\theta_M}$.

If the one-instanton solution had not been discovered analytically more than 20 years ago [12], it would have been found much later by means of the lattice calculations as a constituent of multi-instanton configurations. We suggest that the ephemeron solutions can also be found from the lattice calculations if the periodic boundary conditions are dropped and special attention is paid to the "corner" between the x^+xy- and the x^-xy-hyperplanes. The problem must be posed in the gauge $A^\tau = 0$, and the boundary condition $A_\eta(\tau = 0) = 0$ must be imposed on the gluon field. This type of lattice simulations would immediately account for the interactions between ephemerons, ephemerons and instantons, etc.

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